Int. J. Heat Mass Transfer. Vol. 32, No. 4, pp. 767–774, 1989
Printed in Great Britain

0017-9310/89 $3.00+0.00

Prediction of fluid flow and heat transfer characteristics of turbulent shear flows with a two-fluid model of turbulence

O. J. ILEGBUSI
Department of Materials Science and Engineering, Massachusetts Institute of Technology, Cambridge, MA 02139, U.S.A.

and

D. B. SPALDING
Computational Fluid Dynamics Unit, Imperial College of Science and Technology, London SW7 2AZ, U.K.

(*Received* 15 *September* 1987 *and in final form* 9 *June* 1988)

Abstract—A two-fluid model of turbulence is employed to calculate velocity and temperature characteristics of turbulent boundary layer flows over a flat plate and in heated plane and axisymmetric jets. Emphasis is placed on establishing a model for inter-fluid heat conduction and comparing the results with established data where available.

1. INTRODUCTION

RECENT advances in digital computers have stimulated development of many models for calculating turbulent flows. In a recent work, Markatos [1] has provided an extensive review of such models ranging from the mixing length to the more sophisticated Reynolds' stress closures. The most widely used of these are the single-point closures such as the k–ε model originally proposed by Harlow and Nakayama [2].

While these models have served as valuable tools for simulating practical flows and heat transfer, their performance is far from completely satisfactory when applied to a class of flow in which pressure-induced turbulence is significant. These flows are characterized by the presence of 'structured diffusion' or 'sifting' phenomena such as the formation and growth of large structures and the creation and stretching of small-scale structures. Such situations arise in all swirling flows, Rayleigh–Taylor type flow situations including Kelvin–Helmhotz instabilities, and Taylor and Goertler vortices.

An underlying cause of this inadequacy is that conventional models are based on the unstructured diffusion concepts of Boussinesq [3] and Prandtl [4] in which shear stress is the sole mechanism of turbulence formation. Nevertheless, it is now becoming increasingly evident that turbulence resulting from interaction between body forces and pressure gradients is equally prevalent.

The basic features of the sifting phenomena mentioned earlier have been incorporated into a 'two-fluid' model recently proposed by Spalding [5–8]. In this model, a turbulent fluid is assumed to behave like a two-phase mixture with imaginary interfaces along which there is no surface tension. These two fluids are distinguished only by a difference in their velocity components in the body-force direction. This approach allows the expressions for the various turbulent fluxes to be derived from improved ideas of Prandtl [4]. Thus, the model employs an equation system comprising two sets of velocities, volume fractions (or existence probabilities), temperature and other desirable scalar quantities.

In two earlier papers [9, 10], the present authors formulated the model in terms of the governing equations and the auxiliary relations for inter-fluid friction, entrainment and the associated constants for isothermal systems. The present work represents an extension of the model to heat transfer calculation. In particular, a model is described which expresses conduction of heat between the fluids and the associated constant is optimized by reference to mean-flow data.

The flow situations considered here are relatively simple and well established. They are nevertheless investigated to test the consistency and novelties of the model. Work is being carried out on more complicated flow situations involving pressure gradients, that would provide a stiffer test of the model.

In the following section, the mathematical framework is established. Section 3 contains the computational details while the results are presented in Section 4 and discussed in Section 5. Finally, Section 6 contains the concluding remarks.

NOMENCLATURE

c_p	specific heat at constant pressure
E_{ij}	volumetric entrainment rate of fluid j by fluid i
F_{ij}	volumetric inter-fluid friction
I	intermittency factor
l	length scale
p	static pressure
P_J	sub-layer resistance factor
q_w	heat flux at the wall
Q_{ij}	heat transfer by conduction at the interface of fluids
r	volume fraction
s	dimensionless wall shear stress
St	local Stanton number
T	temperature
u_+	dimensionless velocity near wall
V	generalized velocity
$\overline{vT}$	turbulent heat flux
$\overline{vw}$	turbulent shear stress
w	velocity component in the main-flow direction
w_G	freestream velocity
x	generalized coordinate
y	cross-stream coordinate
y_e	width of calculation domain
y_0	grid width of jets at inlet
z	coordinate in main-stream direction.

Greek symbols

Γ	diffusion flux coefficient
δ	momentum boundary layer thickness
δ_T	thermal boundary layer thickness
θ	dimensionless temperature
κ	von Karman constants
μ	viscosity
ϕ	generic flow variable
ρ	fluid density
σ	Prandtl number
τ_w	shear stress at the wall.

2. MATHEMATICAL FORMULATION

2.1. *Governing equations*

Consider a system of two fluids characterized principally by a difference in the velocities v in the body-force direction. Suppose in addition, that all fragments belonging to one fluid have the same temperature, specie concentration, etc. and that the two fluids share space such that

$$r_1 + r_2 = 1 \tag{1}$$

where r_1, r_2 are their volume fractions or 'existence probabilities'.

The transport of a generic variable ϕ_i (for fluid i) can be expressed by the differential equation

$$\frac{\partial}{\partial t}\rho_i\phi_i + \operatorname{div}(r_i\rho_i V_i\phi_i - \Gamma_{\phi i}\nabla\phi_i) = S_{\phi i}^{ii} + S_{\phi i}^{ij} \tag{2}$$

in which $\Gamma_{\phi i}$ is the 'exchange coefficient', $S_{\phi i}^{ii}$ an intra-fluid source term such as that resulting from pressure gradients, body forces, velocity gradients, etc. and $S_{\phi i}^{ij}$ is an inter-fluid source term due to entrainment of one fluid by the other, friction and heat conduction at the interface. The equation for mass conservation is obtained by setting ϕ_i to unity in equation (2).

Details of the derivation of the expressions for $\Gamma_{\phi i}$, $S_{\phi i}^{ii}$ and $S_{\phi i}^{ij}$ are contained in refs. [5–9] and will not be repeated here. However, a summary of the appropriate expressions is given in Table 1. It should be stressed that in equation (1), Table 1 and subsequent sections, subscripts i and j refer to the fluids and no tensorial notation is implied.

In Table 1, F_B is the body force while G_i is a source term due to velocity gradients which accounts for the tendency of a shear layer to break up into a succession of eddies. This term is negligible for the main-stream momentum equation, but takes the following form for the cross-stream momentum equation:

$$G_i = c_v F_{ij}\left|\frac{\partial w}{\partial y}\right| \tag{3}$$

where w is the mean streamwise velocity.

The expressions for the mass balance and momentum equations were established in refs. [9, 10] while the term Q_{ij} which expresses inter-facial heat conduction is established here for the first time.

Table 1. Diffusion-flux coefficients and source terms

Equation	$\Gamma_{\phi i}$	$S_{\phi i}^{ii}$	$S_{\phi i}^{ij}$
Mass balance	0	0	E_{ij}
Momentum	$c_t l r_i r_j \lvert V_i - V_j \rvert$	$-r_i(\partial p/\partial x l) + F_B + G_i$	$F_{ij} + U_j E_{ij}$
Energy	$c_t l r_i r_j \lvert V_i - V_j \rvert/\sigma_t$	0	$Q_{ij} + T_j E_{ij}$

2.2. *Auxiliary relations*

2.2.1. *Interfacial heat conduction*, Q_{ij}. The allowance for difference in temperature between the two fluids entails heat conduction across the interface. In order to formulate an expression for this heat flux, it is useful to invoke an analogy with inter-fluid momentum flux (i.e. friction).

A postulate for the latter has been discussed in ref. [5] and employed with some success in refs. [9, 10]. A corresponding expression for Q_{ij}, the heat conduction from fluid j to fluid i across the interface is

$$Q_{ij} = c_h c_p \rho_i l^{-1} r_i r_j |w_i - w_j| (T_j - T_i) \quad (4)$$

where $|w_i - w_j|$ is the characteristic 'slip' velocity with which the individual fluid temperatures are transported to the interface, $(T_j - T_i)$ expresses the local fluctuations in temperature, c_p the specific heat and c_h an empirical constant, the determination of which is a main contribution of the present work. It is worth stressing that the form of equation (4) ensures heat is imparted to the cooler fluid at the expense of the hotter fluid.

2.2.2. *Other relations*. As in the earlier work by the authors [9, 10], the entrainment rate E_{ij} of fluid j by fluid i, inter-fluid friction F_{ij} exerted by fluid j on fluid i, and length scale are expressed respectively as

$$E_{ij} = c_m \rho_i (r_j - 0.5) l^{-1} r_i r_j (w_i - w_j) \quad (5)$$

$$F_{ij} = c_f \rho_i l^{-1} r_i r_j |w_i - w_j| (w_j - w_i) \quad (6)$$

$$l = \kappa y \quad \text{for } y/\delta < 0.207 \quad (7)$$

$$l = 0.09\kappa \quad \text{for } y/\delta > 0.207. \quad (8)$$

Equations (7) and (8) are applicable to the flat plate flows. The expression used for the free flows is

$$l = \lambda z \quad (9)$$

where z is the distance from the inlet nozzle along the stream direction and λ deduced from established data for λ/δ and δ/z, δ being the half-width of the jet. The values thus deduced for λ are 0.025 and 0.018 for the plane jet and round jet, respectively.

2.3. *Turbulence model constants*

The set of values of the constants employed here is given in Table 2. All values, except those for σ_t, the Prandtl number, and c_h, the inter-fluid heat conduction constant, were established by the authors in an earlier work [9, 10]. The turbulent Prandtl number should be of the order of 1 and indeed, this value is employed here. This approach ensures that any necessary changes would be accommodated by the value of c_h.

Table 2. Values of the turbulence model constants

c_m	c_v	c_d	c_f	c_t	σ_t	c_h
10.0	0.3	1.0	0.05	10.0	1.0	0.05

2.4. *Boundary conditions at the wall (flat plate)*

Wall friction, τ_s is applied to the near-wall fluid at the first computational cell adjacent to the wall such that

$$\tau_s/\rho w_{pi}^2 = \left\{ \kappa \Big/ \ln \left[\frac{Ey(\tau_s/\rho)^{1/2}}{\mu} \right] \right\}^2 \quad (10)$$

where w_{pi} is the main velocity of fluid i near the wall.

The heat flux from the isothermal surface to this near-wall fluid is deduced from the following expressions established in ref. [11]:

$$\tau_s = \rho^2 c_p w_{pi} St(T_s - T_{pi}) \quad (11)$$

where

$$St = s/\sigma_T (1 + P_J s^{1/2}) \quad (12)$$

$$s = \tau_s/\rho w_{pi}^2 \quad (13)$$

and

$$P_J = 9.0 \left(\frac{\sigma}{\sigma_T} - 1 \right) \left(\frac{\sigma_T}{\sigma} \right)^{1/4} \quad (14)$$

where P_J is a sublayer resistance factor according to Jayatillaka [12], T_{pi} the temperature at the first grid node adjacent to the wall, σ the laminar Prandtl number and σ_T the turbulent Prandtl number.

3. COMPUTATIONAL DETAILS

3.1. *Boundary conditions of particular problems*

Arbitrary volume fractions, temperature and velocity are specified at the start of the integration domain. At the outer boundary, constant fluid 2 velocity and temperature are specified to equal the measured free-stream values. A fixed pressure condition is also imposed at this outer boundary so that entrainment of fluid 2 across it is calculated from continuity.

3.2. *Grid*

Non-uniform grids are employed in all cases with grid densities concentrated near the wall for the flat plate, and near the edges of the jets. The grid in all cases is allowed to expand with shear-layer thickness such that the cross-stream width y_e is expressed as

$$y_e = Az^{0.85} \quad (15)$$

for the flat plate, and

$$y_e/y_0 = a + bz \quad (16)$$

for the jets, where z is the streamwise distance from the inlet, y_0 the value of y_e at the inlet, and A, a and b are empirical constants the values of which are estimated from established rates of growth of the shear layers. The values adopted for these constants are given in Table 3.

The number of cross-stream grid nodes employed for the flat plate, plane jet and round jet are 35, 30 and 40, respectively. These values were selected after

Table 3. Constants of expanding grid formulas for jets

Flow type	a	b
Plane jet	1.0	16.67
Round jet	1.0	15.33

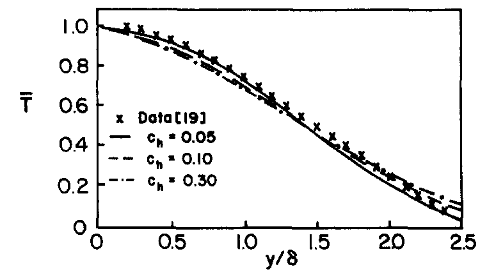

FIG. 2. Effect of c_h on mean temperature similarity profile for plane jet.

systematic grid refinement tests. In addition, the forward-step size is limited to 15% of the cross-stream width in all cases.

3.3. *Solution procedure*

The parabolic system of equations coupled with the boundary conditions described above are solved by the forward-marching, iterative finite-domain solution procedure embodied in the PHOENICS [13] computer code. This code employes a derivative of the SIMPLE algorithm of ref. [14].

4. RESULTS

4.1. *Optimization of the model constant* c_h

The present model does not specifically define the two fluids as turbulent/nonturbulent; thus, the value of c_h cannot strictly be deduced by comparing predictions with conditionally-sampled data such as those reported in ref. [15]. However, these data can at least indicate the order of magnitude of this constant since they offer the closest experimental approximation available to the two-fluid concept.

By examining such data, it can be deduced that c_h should be of the order of 0.1 after equating the expression in equation (4) to the local excess flux of heat in the irrotational zone. This is indeed the value about which optimization is performed in the present work.

The results of this investigation are shown in Figs. 1 and 2 for flat plate and plane jet, respectively. Also shown are the experimental data of Reynolds *et al.* [16] in Fig. 1 and those of Zijnen [17] for the plane jet in Fig. 2. It is seen that the mean temperatures are highly sensitive to the values of c_h in the near-wall region of the flat plate and in the entire jet. It is also apparent that a value of $c_h = 0.05$ best approximates the measurements in both cases. This values is chosen for the calculations reported here.

4.2. *Predictions*

4.2.1. *Flat plate boundary layer.* Figure 3 shows the predicted similarity profiles of mean temperature compared with the data of Reynolds *et al.* [16]. The mean temperature has been calculated from the following general relationship:

$$\phi = r_1\phi_1 + r_2\phi_2. \tag{17}$$

The cross-stream distance in this figure has been nondimensionalized by the thermal boundary layer thickness, defined at a location where the mean fluid temperature attains 99.95% of the free-stream value. It is seen that the mean temperature appears to be well simulated.

Figure 4 displays the profiles of the individual fluid temperatures across the boundary layer. This figure shows that fluid 1 which is directly heated by the wall is generally hotter than fluid 2. The difference between the two fluid temperatures decreases towards the outer boundary as the colder fluid 2 is continuously entrained.

The predicted mean velocity and individual velocity similarity profiles are shown in Figs. 5 and 6, respectively. Also given in Fig. 5 are the experimental data of Leslie *et al.* [18]. In each figure, the cross-stream distance has been normalized by the momentum boundary layer thickness. The predicted mean velocities appear to agree well with the data.

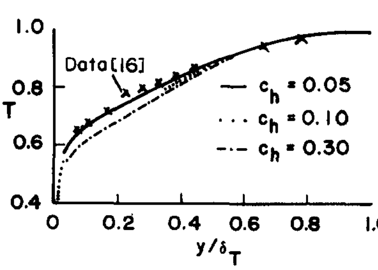

FIG. 1. Effect of c_h on mean temperature similarity profile for flat plate.

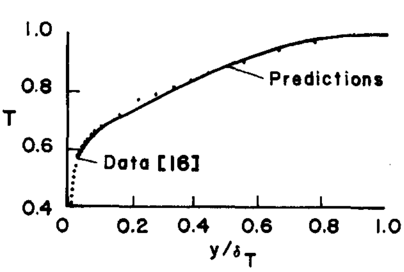

FIG. 3. Mean temperature similarity profile for flat plate.

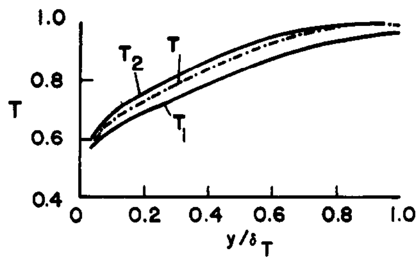

FIG. 4. Predicted fluid temperature profiles for flat plate.

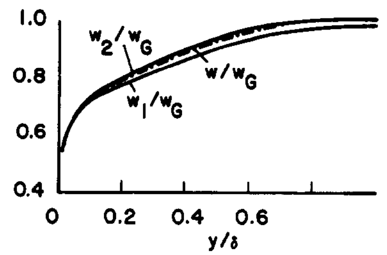

FIG. 6. Predicted fluid velocity similarity profiles for flat plate.

As discussed in ref. [9], the intermittency factor I, can be defined in relation to the symmetry expression contained in the entrainment equation (5) as

$$I = \min(1.0, 2r_1) \tag{18}$$

where r_1 is the volume fraction of the near-wall fluid. The predicted similarity profile of I is compared with the conditionally-sampled data of Leslie *et al.* [18] in Fig. 7. It should be re-emphasized here that these authors specifically defined the two fluids as turbulent/nonturbulent while the present model does not employ such a restriction. It is seen that the predicted intermittency factor, though in the consensus of the data, is generally underpredicted and the discrepancy is more pronounced in the inner region containing the turbulent fluid. Clearly, this behavior may be attributed to the difference in characterization of the two fluids between the model and the experimental investigation.

Figure 8 shows the predicted dimensionless entrainment rate. There is no entrainment at the wall and in the free-stream which consist entirely of single fluids. The entrainment rate however attains a maximum value at about 73% of the boundary layer thickness from the wall. This value compares with the location of maximum 'crossing frequency' in ref. [18].

The predicted local heat transfer coefficient (Stanton number) at the fully-developed region is compared with the data of Reynolds *et al.* [16] in Fig. 9. The abscissa in this figure is the Reynolds number based on the axial distance along the flat plate. The agreement between predictions and data is satisfactory in the fully-developed region. The discrepancy in the developing region is probably attributable to the boundary conditions specified at the inlet of the calculation domain.

4.2.2. *Plane jet and round jet.* In Table 4, the predicted rates of spread, maximum shear stress, maximum turbulent heat flux and maximum entrainment rate are compared with the values deduced from the measurements on the one hand, and those often quoted for the k–ε model of turbulence on the other. The exact expressions for turbulent fluxes have been derived in ref. [5] and are of the general form

$$v\phi = r_1 r_2 (v_1 - v_2)(\phi_1 - \phi_2). \tag{19}$$

It should be remarked that the values shown as the measured data were actually deduced by the present authors from the mean flow measurements because the values obtained by direct measurement did not close the mean thermal balance.

Table 4 shows that the predictions are generally in good agreement with the experimental data. It is also observed that the rate of spread of heat is much larger than that of momentum. In addition, the results obtained with the present model are generally comparable to those reported for the more popular k–ε model. It should be noted that the k–ε values for the round jet were obtained after modifying the model constants in line with the suggestion of Rodi [19].

A comparison of the results in terms of the cross-stream similarity profiles of the mean and turbulence

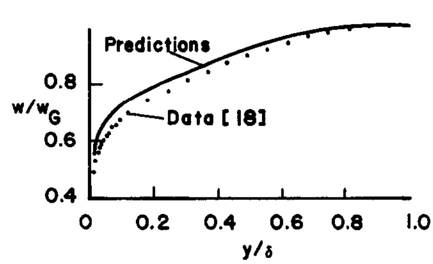

FIG. 5. Mean velocity similarity profiles for flat plate.

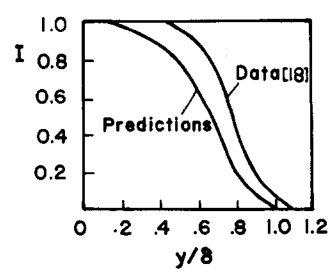

FIG. 7. Intermittency factor similarity profile for flat plate.

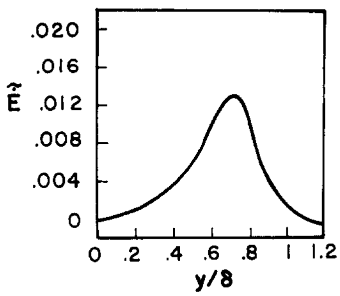

FIG. 8. Predicted similarity profile of entrainment rate for flat plate.

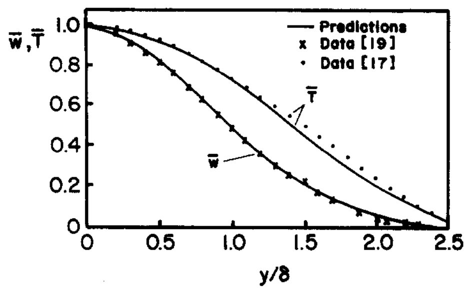

FIG. 10. Mean temperature and velocity similarity profiles for plane jet.

quantities are presented in Figs. 10–15. Each figure includes the predictions and experimental data. Figures 10 and 11 show the mean temperature and velocity similarity profiles for the plane jet and round jet, respectively. These figures show that the widths of the temperature similarity profiles are larger than those of the velocity profiles. The agreements between predictions and measurements appear to be satisfactory. However, in the plane jet (Fig. 10), the predicted approach to the external temperature value is faster than the data while the opposite trend is observed for the velocity profile.

Figures 12 and 13 show the measured mean and conditioned temperatures and the predicted mean and individual fluid temperatures. The predicted individual fluid temperatures are in the consensus of the measurements notwithstanding the difference in the definitions of the fluids.

Finally, Figs. 14 and 15 show the predicted and measured similarity profiles of the turbulent shear stress and heat fluxes for the plane and round jets, respectively. The agreement in both cases could be considered satisfactory especially in the core regions of the flows. The results at the outer region are however less satisfactory.

5. DISCUSSION

The results presented above have shown that the mean flow and temperature characteristics of turbulent shear layers can be reasonably well simulated with the two-fluid model of turbulence.

The value of 0.05 obtained for the inter-fluid diffusion heat transfer coefficient is of the same order of magnitude as (but not greater than) that for momentum. It is not surprising that this value differs

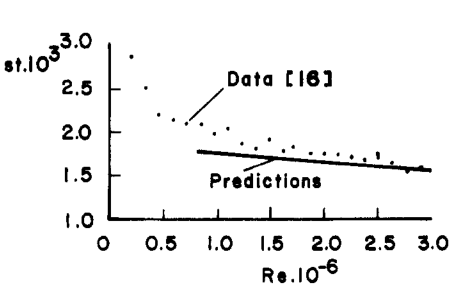

FIG. 9. Local heat transfer coefficient along flat plate.

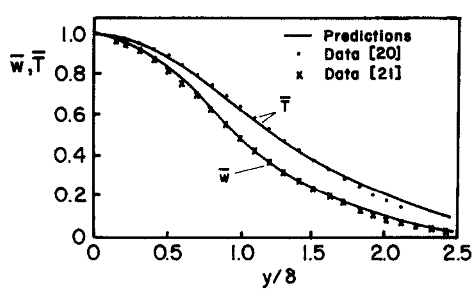

FIG. 11. Mean temperature and velocity similarity profiles for round jet.

Table 4. Calculated and measured characteristics of self-preserving jets

Quantity	Plane jet Data	k–ε	Two fluid	Round jet Data	k–ε	Two fluid
$d\delta/dz$	0.110	0.104	0.120	0.086	0.085	0.087
$d\delta_T/dz$	0.14	0.150	0.145	0.11	0.102	0.105
$\overline{vw}_m$	0.024	0.021	0.020	0.019	0.016	0.014
$\overline{vT}_m$	0.028	0.028	0.029	0.021	0.019	0.120
E_m	—	—	0.06	0.051	—	0.050

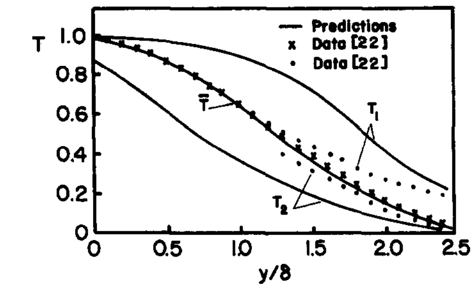

FIG. 12. Predicted mean and individual fluid temperature profiles compared with measured mean and conditioned temperature profiles for plane jet.

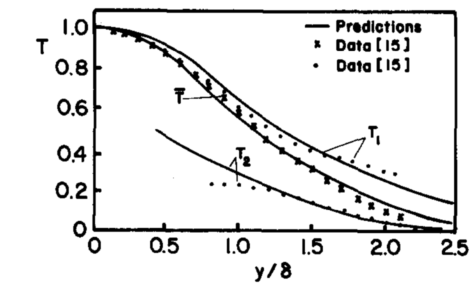

FIG. 13. Predicted mean and individual fluid temperature profiles compared with measured mean and conditioned temperature profiles for round jet.

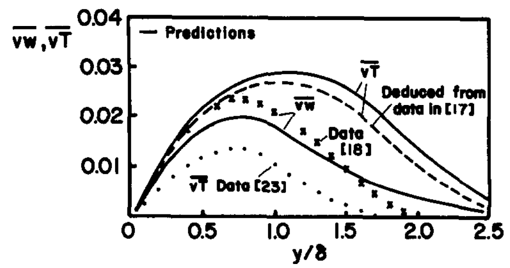

FIG. 14. Predicted and measured shear stress similarity profiles for plane jet.

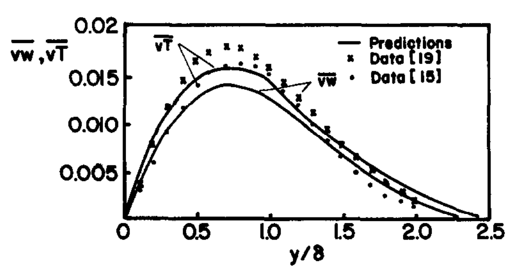

FIG. 15. Predicted and measured shear stress similarity profiles for round jet.

from that deduced from conditional sampling data. The latter specifically defines the two fluids as turbulent/nonturbulent while the present model distinguishes them by the difference in their cross-stream velocity components. This distinction is reflected through the whole calculation as evident in the comparison of the individual fluid properties with the conditionally-sampled data.

Of special interest is that the same set of constants was used for all predictions, including the round jet, which has traditionally required modification of some constants of conventional turbulence models. In addition, predictions of mean flow characteristics including the heat transfer coefficient at the wall appear to be as good as those obtained by other workers with the more popular k–ε model.

Some of the unacceptable results such as the predicted heat flux in the free shear layers could conceivably be improved upon by adjusting the model constants. However, the effects on the other results would need to be evaluated.

Of course, the large number of constants in the model is a drawback. But since the expressions with which they are associated have physical basis, a set of values such as those in Table 1 that can predict mean flow characteristics reasonably well will probably suffice for practical flow simulation.

This work is a small step in the long road to establishing the two-fluid model as a viable tool. A stiffer test demands its application to more complex flow situations including those with significant pressure gradients. This aspect will be the subject of the next investigation.

6. CONCLUSION

A two-fluid model of turbulence has been used to calculate fluid flow and heat transfer characteristics of turbulent shear layers including flat-plate boundary layer, a plane jet and a round jet. A model is formulated to represent conduction of heat at the interface of the constituent fluids and the associated constant in this model is deduced by reference to available experimental data. The same set of constants is employed for all flows and the model predictions of mean-flow characteristics agree satisfactorily with the experimental data.

Further work is being planned to apply the model to more complex flow situations such as those involving significant pressure gradients.

Acknowledgements—This work was initiated at the Computational Fluid Dynamics Unit of Imperial College, London, and completed at the Mathematical Modelling Center of the Department of Materials Science and Engineering, Massachusetts Institute of Technology, Cambridge, Massachusetts, U.S.A.

REFERENCES

1. N. C. Markatos, The mathematical modelling of turbulent flows, *Appl. Math. Modelling* **10**, 190 (1986).

2. F. H. Harlow and P. I. Nakayama, Turbulent transport equations, *Physics Fluids* **10,** 2323 (1967).
3. J. Boussinesq, Theorie de l'ecoulement tourbillant, *Mem. Acad. Sci.* **23,** 46 (1877).
4. L. Prandtl, Bericht uber untersuchungen Zur ausgebildeten Turbulenz, *ZAMM* **5,** 136 (1925).
5. D. B. Spalding, Two-fluid models of turbulence, NASA Langley Workshop on Theoretical Approaches to Turbulence, Hampton, Virginia (1984).
6. D. B. Spalding, Chemical reaction in turbulent fluids, *J. PhysicoChem. Hydrodyn.* **4**(4), 323 (1982).
7. D. B. Spalding, Towards a two-fluid model of turbulent combustion in gases with special reference to the spark ignition engine, I. Mech. E. Conf., Oxford, April 1983. In *Combustion in Engineering*, Vol. 1, Paper No. C53/83, pp. 135–142 (1983).
8. D. B. Spalding, Turbulence modelling: a state-of-the-art review, Imperial College, CFDU Report CFD/83/3 (1983).
9. O. J. Ilegbusi and D. B. Spalding, A two-fluid model of turbulence and its application to near-wall flows, *PCH PhysicoChem. Hydrodyn.* **9**(1/2), 127 (1987).
10. O. J. Ilegbusi and D. B. Spalding, Application of a two-fluid model of turbulence to turbulent flows in conduits and free shear layers, *PCH PhysicoChem. Hydrodyn.* **9**(1/2), 161 (1987).
11. S. W. Chi and D. B. Spalding, Influence of temperature ratio on heat transfer to a flat plate through a turbulent boundary layer in air, *Proc. 3rd Int. Heat Transfer Conf.*, Chicago (AIChE), Vol. II, pp. 41–49 (1966).
12. C. L. K. Jayatillaka, The influence of Prandtl number and surface roughness on the resistance of the laminar sub-layer to momentum and heat transfer, Ph.D. Thesis, University of London (1966).
13. D. B. Spalding, A general purpose computer program for multi-dimensional one- and two-phase flows, *J. Math. Comput. Simulation* **XXIII,** 267 (1981).
14. S. V. Patankar and D. B. Spalding, A calculation procedure for heat, mass and momentum transfer in three-dimensional parabolic flows, *Int. J. Heat Mass Transfer* **15,** 1787 (1972).
15. R. Chevray and N. K. Tutu, Intermittency and preferential transport of heat in a round jet, *J. Fluid Mech.* **88**(1), 133 (1978).
16. W. C. Reynolds, W. M. Kays and S. J. Kline, Heat transfer in the turbulent incompressible boundary layer I—constant wall temperature, NACA Tn-33-1 (1958).
17. B. G. van der Hegge Zijnen, Measurement of the distribution of heat and matter in a plane turbulent jet of air, *Appl. Scient. Res.* **4,** 405 (1958).
18. S. G. Leslie, V. K. Kovasznay and R. F. Blackwelder, Large-scale motion in the intermittent region of a turbulent boundary layer, *J. Fluid Mech.* **41**(2), 283 (1970).
19. W. Rodi, The prediction of free turbulent boundary layers by the use of two-equation model of turbulence, Ph.D. Thesis, University of London (1972).
20. W. Rodi, A review of experimental data on free turbulent boundary layers. In *Studies in Convection, Theory, Measurements and Applications* (Edited by B. E. Launder). Academic Press, London (1975).
21. C. J. Chen and W. Rodi, Vertical turbulent buoyant jets—a review of experimental data. In *Heat and Mass Transfer*, Vol. 4. Pergamon Press, Oxford (1980).
22. I. Wygnanski and H. Fiedler, Some measurements in the self-preserving jet, *J. Fluid Mech.* **38,** 599 (1969).
23. A. E. Davies, J. F. Keffer and W. D. Baines, Spread of a heated plane turbulent jet, *Physics Fluids* **18**(7), 770 (1975).
24. J. Bashir and M. S. Uberoi, Experiments on turbulent structure and heat transfer in a two-dimensional jet, *Physics Fluids* **18**(4), 405 (1975).

PREDICTION DES CARACTERISTIQUES DYNAMIQUES ET THERMIQUES DES ECOULEMENTS TURBULENTS CISAILLANTS AVEC UN MODELE DE TURBULENCE A DEUX FLUIDES

Résumé—Un modèle de turbulence à deux fluides est utilisé pour calculer les caractéristiques de vitesse et de température des écoulements à couche limite turbulente sur une plaque plane et dans des jets chauds plans ou axisymétriques. L'attention est portée sur l'établissement d'un modèle pour rendre compte de la conduction interfaciale et dans la comparaison des résultats avec des données existantes valables.

BERECHNUNG DER STRÖMUNG UND DES WÄRMEÜBERGANGS IN TURBULENTEN SCHERSTRÖMUNGEN MIT EINEM ZWEIFLUID-TURBULENZ-MODELL

Zusammenfassung—Mit Hilfe eines Zweifluid-Turbulenz-Modells wird die Geschwindigkeits- und Temperaturverteilung einer turbulenten Grenzschichtsströmung über einer ebenen Platte und in beheizten ebenen und achsensymmetrischen Strahlen berechnet. Besonders ein Modell für die Wärmeleitung im Fluid wird hier mit einbezogen. Die Ergebnisse werden mit experimentellen Daten verglichen—soweit vorhanden.

РАСЧЕТ ХАРАКТЕРИСТИК ТЕЧЕНИЯ ЖИДКОСТИ И ТЕПЛОПЕРЕНОСА В ТУРБУЛЕНТНЫХ—СДВИГОВ Х ПОТОКАХ НА ОСНОВЕ ДВУХЖИДКОСТНОЙ МОДЕЛИ ТУРБУЛЕНТНОСТИ

Аннотация—Для расчета скоростных и температурных характеристик турбулентных течений в пограничном слое на плоской пластине и в нагретых плоских и осесимметричных струях используется двухжидкостная модель турбулентности. Особое внимание уделяется разработке модели теплопроводности между жидкостями, а также сравнению результатов с расчетными данными.